RESEARCH CENTRE

Inria Saclay Centre at Université Paris-Saclay

IN PARTNERSHIP WITH:

Université Paris-Saclay

2024

ACTIVITY REPORT

Project-Team

QUACS

Quantum Computation Structures

IN COLLABORATION WITH: Laboratoire de Méthodes Formelles

DOMAIN

Algorithmics, Programming, Software and Architecture

THEME

Proofs and Verification

Contents

Project-Team QUACS

Creation of the Project-Team: 2021 December 01

Keywords

Computer sciences and digital sciences

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A2.1.1. – Semantics of programming languages

A2.2.1. – Static analysis

A2.4. – Formal method for verification, reliability, certification

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A7.3.1. – Computational models and calculability

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A8.6. – Information theory

A8.7. – Graph theory

Other research topics and application domains

B5.11. – Quantum systems

1 Team members, visitors, external collaborators

Research Scientists

- Pablo Arrighi [Team leader, UNIV PARIS SACLAY, Professor Detachement, until Sep 2024]
- Pablo Arrighi [Team leader, UNIV PARIS SACLAY, Senior Researcher, from Oct 2024]
- Pablo Arnault [INRIA, Researcher, Adjunct Professor at UNIV PARIS SACLAY]
- Renaud Vilmart [INRIA, ISFP]
- Vladimir Zamdzhiev [INRIA, ISFP]
- Marc de Visme [INRIA, Researcher]

Faculty Members

- Marc Baboulin [UNIV PARIS SACLAY, Professor, from Feb 2024, HDR]
- Benoît Valiron [CENTRALE, Associate Professor]
- Matt Wilson [CENTRALE, Associate Professor, from Aug 2024]

Post-Doctoral Fellows

- Alexandre Clement [INRIA, Post-Doctoral Fellow]
- Cole Comfort [INRIA, Post-Doctoral Fellow, from Dec 2024]
- James Hefford [INRIA, Post-Doctoral Fellow]
- Marco John Lewis [INRIA, from Nov 2024]
- Nesta J Judah Van Der Schaaf [INRIA, Post-Doctoral Fellow, from Nov 2024]

PhD Students

- Dogukan Bakircioglu [INRIA]
- Marin Costes [ENS PARIS-SACLAY]
- Kinnari Dave [INRIA Nancy]
- Nicolas Heurtel [Quandela]
- Océane Koska [Eviden]
- Julien Lamiroy [UNIV PARIS SACLAY]
- Louis Lemonnier [ENS PARIS-SACLAY, until Jan 2024]
- Thea Li [INRIA, from Sep 2024]
- Octave Mestoudjian [UNIV PARIS SACLAY]
- Jérome Ricciardi [CEA]
- Matthieu Robeyns [UNIV PARIS SACLAY]
- Adham Zekri [INRIA, from Dec 2024]

Technical Staff

- Tristan Cam [INRIA, Engineer, until Feb 2024]
- Michel Nicolis [INRIA, Engineer, from Nov 2024]

Interns and Apprentices

- Nicolas Dumange [UNIV PARIS SACLAY, from Feb 2024 until Jul 2024]
- Antoine Guilmin-Crepon [ENS PARIS-SACLAY, from Feb 2024 until Jul 2024]
- Thea Li [UNIV PARIS SACLAY, from Apr 2024 until Jul 2024]
- Srijita Nandi [INRIA, Intern, from Jun 2024 until Aug 2024]
- Michel Nicolis [UNIV PARIS SACLAY, from Mar 2024 until Sep 2024]
- Seydina Ousmane Kandé [UNIV PARIS SACLAY, from Jun 2024 until Aug 2024]
- Brice Pointal [UNIV PARIS SACLAY, from Mar 2024 until Sep 2024]
- Si Chen Qiao [INRIA, Intern, from May 2024 until Aug 2024]
- Léo Soudant [ENS PARIS-SACLAY, until Jul 2024]
- Panagiotis Theodorakopoulos [UNIV PARIS SACLAY, from May 2024 until Aug 2024]
- Raphael de Saint Albin [UNIV PARIS SACLAY, from Jun 2024 until Jul 2024]

Administrative Assistant

- Joyce Soares Brito [INRIA]

Visiting Scientists

- Alejandro Díaz Caro [University of Buenos Aires, from Nov 2024]
- Vitor Fernandes [Universidade do Minho, from Mar 2024 until Jul 2024]
- Carla Ferradini [ETH Zurich, from Jun 2024 until Jul 2024]
- Masha Sosonkina [Old Dominion University, Norfolk, from Sep 2024 until Oct 2024]

External Collaborator

- Mathieu Nguyen [CNRS, until Sep 2024]

2 Overall objectives

Quantum information processing is one of the rising forces of the information era. Encoding information within quantum systems and manipulating them promises to lead to great advantages, with three main application domains: quantum cryptography, quantum simulation, and quantum algorithmics. To understand its strengths and limits, we take a transversal stance and seek to capture which resources are granted to us by nature, at the fundamental level, for the sake of computing (e.g. quantum and spatial parallelism). We do so by abstracting away physics' ability to compute, into formal models of quantum computation (e.g. quantum automata and graph rewriting models). We then verbalize its main structures as quantum programming languages (e.g. quantum lambda-calculus, process algebra). Actually, the process goes both ways, when developments in quantum programming languages lead to the discovery of new structures which may or may not be compilable into formal models of quantum computation, raising the sometimes fascinating question of the physicality of these resources.

3 Research program

3.1 Quantum simulation

One usually distinguishes three main fields of applications of Quantum Computing: quantum cryptography (short-term), quantum simulation (mid-term), quantum algorithmic (long-term). Quantum simulation then divides into two subfields: continuous-time quantum simulation, which is very physicky and consists of ad hoc emulation of one Hamiltonian by another, and discrete-time quantum simulation, which is much closer to quantum algorithmic: this is where we stand. In particular, we focus on the provision of a quantum-circuit description of the dynamics of fundamental particles. In particular, as we design these quantum simulation schemes, our focus is on retaining the symmetries of the simulated model. This is both a matter of efficiency and correctness. For instance, our discretizations have a maximum speed of propagation of the information, which coincides with the speed of light in the simulated system, as a first step towards retaining Lorentz symmetry. Similarly, our discretizations exhibit the gauge symmetries that motivate the different fundamental particles. The long term goal of this program is to provide a satisfactory quantum-circuit descriptions of the whole standard model of particle physics.

3.2 Semantics and Programming

In the research program on Semantics, the QuaCS team is working on developing mathematical methods and tools that formulate the precise meaning and behavior of (quantum) systems, processes, type systems and programming languages, other formal languages and computational models. This includes, but is not limited, to the following:

- Operational semantics: a mathematically precise description of the dynamics of quantum programs and other computational models (e.g., the small-step semantics of quantum lambda calculi, token-machine semantics of quantum diagrammatic calculi).

- Mathematical and denotational semantics: a mathematical interpretation of a quantum programming language, process theory, diagrammatic calculus, etc., which is always expected to be sound and often expected to be adequate or complete.

This line of research is focused on identifying fundamental connections between the static specification (e.g. syntax) of quantum languages, their dynamic behavior (e.g. operational semantics) and their mathematical interpretation (e.g. denotational semantics) with the intention of developing each of these components further.

3.3 Languages and algorithms for quantum computation

The QuaCS team is involved in the development and study of graphical calculi such as quanctum circuits, ZX-, ZW-, ZH-calculi, but also languages for linear optics, such as the LOv-calculus. These languages are supposed to represent particular features of quantum computing, and hence are designed with a particular semantics in mind. A question of interest in the field is that of completeness with respect to that semantics : the ability to graphically turn any two equivalent diagrams into one another, making it possible to entirely reason within the language. The team is interested in the structure quantum operators have, that can be exhibited by the graphical approach, and depending on the model of computation at hand. It then becomes possible to study the links between the graphical languages, and hence, between the different models of computation. Recently, some focus has been put in the use of graphical languages for the study of indefinite causal orders, a extension to the usual quantum computation model, where not only data is quantum, but also the control flow of the program, which is allowed by the theory but still not well understood. The QuaCS team also studies quantum algorithms that could be used in concrete scientific applications and in particular High Performance Computing (HPC). Among them we focus on hybrid quantum-classical techniques that can take advantage of the respective strengths of each type of processors. A related activity concerns the efficient simulation of such algorithms on current HPC platforms.

4 Application domains

4.1 Quantum simulation

Feynman's invention of Quantum Computing really came out of a frustration: that of seeing classical computers take such a long time to simulate quantum systems. His intuition was that «quantum computers» would do a better job at simulating quantum systems. There is not the slightest doubt indeed that quantum simulation will have major outcomes for society. Thinking about it, most of the objects that surround us (cars, computers, furniture. . .) are designed on computers, thanks to the fact that we can prototype and simulate them on classical computers. That is, up to a certain scale. Below that we are left in the dark as quantum effects come into play, yielding an exponential blow up of the cost or simulation. For now. But, the day we will have good quantum computers and good quantum simulation algorithms to run upon them, we will be able to simulate these particles, atoms, molecules and the way they interact. Consequently we will be able to design specific-purpose molecules, materials, nanotechnologies, with applications in chemistry, biochemistry, electronics, mechanics. At QuaCS we focus on the bottom layer: the quantum-simulation algorithms for fundamental particles. After all, to be able to efficiently simulate fundamental interactions is to be able to simulate virtually everything, from first principles. An added bonus of this strand of research is that usually when we express some physics as a quantum algorithm, it becomes way simpler, more explanatory.

4.2 Semantics and Programming

This line of research can reveal interesting connections between mathematical structures, computational models, type systems and other formal languages. Ideally, one endpoint of such a connection can be used to influence the design and development of the other endpoint, because these connections can allow us to improve our understanding of the different aspects of the (quantum) systems and computational models under consideration.

For instance, monads in category theory were the inspiration for introducing monads in programming languages. Another example includes categorical quantum mechanics which lead to the development of the ZX-calculus along with other useful tools, such as PyZX/QuiZX, which may be used for optimisation of quantum circuits and classical simulation of quantum processes.

4.3 Languages and algorithms for quantum computations

One of the main features of graphical languages is that they can be made abstract enough to remove unnecessary clutter and ease reasoning on quantum operators. This has several consequences : They are rather intuitive to work with, while at the same time being completely formal They can provide an intermediate representation of quantum programs, with enough abstraction to reason about and modify the program during compilation. The most illustrative example of such modification is circuit optimisation, where the goal is to reduce the number of "expensive" quantum gates in the circuit, which can be achieved by turning the circuit into a ZX-diagram, then using its equational theory to perform the reduction. Together with the simplification heuristic, it is possible to exploit this "uncluttering" effect to perform more efficient classical simulation of quantum programs. It can be exploited to perform automated verification of quantum programs. The hybrid quantum-classical algorithms that have been developed in the team concern more specifically the solution of linear systems, which is a key function for most scientific and data science applications. For example we are developing a new solver that combines the Quantum Singular Value Transformation with iterative improvement to produce accurate linear system solutions.

5 New results

5.1 Minimal equational theory for the d-dimensional ZW calculus

Participants: Renaud Vilmart, Marc de Visme.

[18] *To appear in the proceedings of Conference on Computer Science Logic CSL 2025, Feb 2025, Amsterdam, Netherlands.*

We designed a graphical language for quantum computation in dimension d higher than 2. While this is not the first work in this direction, by focussing on the ZW calculus and its combinatorics properties, we are able to obtain an comparatively simple language with a small and minimal (and complete) equational theory: more precisely, two circuits represent the same quantum map if and only if they can be transformed one into the other using the equations, and each of this equation is proved to be necessary and independent from the others.

Additionally, we extend this work into a language for mixed-dimensional quantum comptuation (i.e. the dimension 'd' is no longer a fixed integer), and provide a minimal and complete equational theory for that extension too.

5.2 Time arrow without past hypothesis : a toy model approach

Participant: Pablo Arrighi.

[6][3] *New J. Phys. 26 113019, doi, (2024). Pre-print arXiv :2306.07121v2.Proceedings of RC 2024. Pre-print arXiv :2306.07121v1.*

The laws of Physics are time-reversible, making no qualitative distinction between the past and the future – yet we can only go towards the future. This apparent contradiction is known as the "arrow of time problem". Its current resolution states that the future is the direction of increasing entropy. But entropy can only increase towards the future if it was low in the past, and past low entropy is a very strong assumption to make, because low entropy states are rather improbable, non-generic. Recent works from the Physics literature suggest, however, we may do away with this so-called "past hypothesis", in the presence of reversible dynamical laws featuring expansion. We prove that this can be the case in principle, within a toy model. It consists in graphs upon which particles circulate and interact according to local reversible rules. Some rules locally shrink or expand the graph. We prove that almost all states expand; entropy always increases as a consequence of expansion – thereby providing a local explanation for the rise of an entropic arrow of time without the need for a past hypothesis. The discrete setting of this toy model allows us to deploy the full rigour of theoretical Computer Science proof techniques.

5.3 Quantum networks theory

Participant: Pablo Arrighi, Amélia Durbec, Matt Wilson.

[4] *Quantum networks theory. 43 pages, Quantum 8, 1508 (2024). Pre-print arXiv :2110.10587.*

The formalism of quantum theory over discrete systems is extended in two significant ways. First, quantum evolutions are generalized to act over entire network configurations, so that nodes may find themselves in a quantum superposition of being connected or not, and be allowed to merge, split and reconnect coherently in a superposition. Second, tensors and traceouts are generalized, so that systems can be partitioned according to almost arbitrary logical predicates in a robust manner. The hereby presented mathematical framework is anchored on solid grounds through numerous lemmas. Indeed, one might have feared that the familiar interrelations between the notions of unitarity, complete positivity, trace-preservation, non-signalling causality, locality and localizability that are standard in quantum theory be jeopardized as the neighbourhood and partitioning between systems become both quantum, dynamical, and logical. Such interrelations in fact carry through.

5.4 Semantics for a Turing-Complete Reversible Programming Language with Inductive Types

Participant: Louis Lemonnier, Benoît Valiron.

[7] *Proceedings of FSCD 2024, doi: 10.4230/LIPIcs.FSCD.2024.19*

This paper is concerned with the expressivity and denotational semantics of a functional higher-order reversible programming language based on Theseus. In this language, pattern-matching is used to ensure the reversibility of functions. We show how one can encode any Reversible Turing Machine in said language. We then build a sound and adequate categorical semantics based on join inverse categories, with additional structures to capture pattern-matching and to interpret inductive types and recursion. We then derive a notion of completeness in the sense that any computable, partial, first-order injective function is the image of a term in the language.

5.5 A tree-approach Pauli decomposition algorithm with application to quantum computing

Participant: Marc Baboulin, Océane Koska.

[20] *Proceedings of ISC High Performance, Hamburg, Germany, pp. 1-11 (2024).*

The Pauli matrices are 2-by-2 matrices that are very useful in quantum computing. They can be used as elementary gates in quantum circuits but also to decompose any matrix as a linear combination of tensor products of the Pauli matrices. However, the computational cost of this decomposition is potentially very expensive since it can be exponential in n. In this paper, we propose an algorithm with a parallel implementation that optimizes this decomposition using a tree approach to avoid redundancy in the computation while using a limited memory footprint. We also explain how some particular matrix structures can be exploited to reduce the number of operations. We provide numerical experiments to evaluate the sequential and parallel performance of our decomposition algorithm and we illustrate how this algorithm can be applied to encode matrices in a quantum memory.

5.6 The Category of Operator Spaces and Complete Contractions

Participant: Vladimir Zamdzhiev.

[14]

We show that the category OS of operator spaces, with complete contractions as morphisms, is locally countably presentable. This result, together with its symmetric monoidal closed structure with respect to the projective tensor product of operator spaces, implies the existence of cofree (cocommutative) coalgebras with respect to the projective tensor product and therefore provides a mathematical model of Intuitionistic Linear Logic in the sense of Lafont.

5.7 Quantum Circuit Completeness: Extensions and Simplifications

Participant: Alexandre Clément, Renaud Vilmart.

[9] *Proceedings of Conference on Computer Science Logic CSL 2024, Feb 2024, Naples, Italy. doi: 10.4230/LIPIcs.CSL.2024.20*

We refine the recent breakthrough paper ([8]) that establishes a completeness result for quantum circuits, a problem that was left open since the avent of quantum computing. The latter shows that a given set of equalities between circuits enirely captures their semantics. The result is refined by 1/ removing unnecessary equations, 2/ simplifying the most complicated equation, 3/ extending it to allow for measurements.

5.8 Efficient Block Encoding of Simple Matrices from Finite Difference Method

Participant: Renaud Vilmart.

[17]

One of the most promising uses of quantum computation is in simulating physical systems, e.g. by accelerating classical numerical methods like the finite difference method. In several of the proposals for solving this problem, one needs to encode a square matrix (describing the physics of the system) into part of a quantum circuit. We show here, for some typical ideal matrices, how to efficiently perform this encoding: the resulting circuit has a logarithmic depth wrt the number of qubits, which itself is logarithmic in the size of the initial matrix.

5.9 Unitary event structures

Participant: Vitor Fernandes, Marc de Visme, Benoît Valiron.

[11]

We study a toy language for concurrent processes in presence of non-determism, probabilities or quantum behaviours. For that, we rely on the pre-existing notion of probabilistic and quantum event structure – a “truly concurrent" approach to representing processes using partial orders – to give a denotational semantics to processes in those three cases. In order to get a closer match between the language and its semantics, we refine the existing notion of quantum event structures by defining unitary event structures.

5.10 A typology of quantum algorithms

Participant: Pablo Arrighi, Pablo Arnault.

[19] We draw the current landscape of quantum algorithms, by classifying about 130 quantum algorithms, according to the fundamental mathematical problems they solve, their real-world applications, the main subroutines they employ, and several other relevant criteria. The primary objectives include revealing trends of algorithms, identifying promising fields for implementations in the NISQ era, and identifying the key algorithmic primitives that power quantum advantage.

5.11 Running position-dependent quantum walks with quantum circuits

Participant: Pablo Arnault.

[15][21]

We provide quantum circuits that enable to run quantum walks that have position-dependent coin operators. Such quantum-walk models have paramount applications, such as (i) modeling quantum particles in force fields/energy potentials, in particular relativistic models, and (ii) modeling quantum particles subject to noise. We provide essentially two types of circuits.

The first type [15] is an approximate circuit that is efficient and that works provided to position dependence of the coin operators is smooth, so typically this could be used for smooth force fields (especially close to the continuum limit), but not for noise models which are sharp even at the scale of the lattice spacing.

The other type of circuits [16][21]are exact circuits that, have a width and depth that can be adjusted at will, depending on the needs of the experimental platform: the circuit having maximum depth and minimum width, has a depth that is simply proportional to the number of positions of the walk, and the width is logarithmic in this number of positions since the position is encoded in base 2 (with n qubits we can encode 2^n positions); then, one can transfer as much of the depth as one wants into the width of the circuit, so that the circuit having minimum depth and maximum width, has a depth that is logarithmic in the number of positions, and a width that is linear in this number (which is the inverse situation with respect to the circuit having maximum depth and minimum width).

5.12 Rewriting and Completeness of Sum-Over-Paths in Dyadic Fragments of Quantum Computing.

Participant: Renaud Vilmart.

[5]

The "Sum-Over-Paths" formalism is a way to symbolically manipulate linear maps that describe quantum systems, and is a tool that is used in formal verification of such systems. We give here a new set of rewrite rules for the formalism, and show that it is complete for "Toffoli-Hadamard", the simplest approximately universal fragment of quantum mechanics. We show that the rewriting is terminating, but not confluent (which is expected from the universality of the fragment). We do so using the connection between Sum-over-Paths and graphical language ZH-calculus, and also show how the axiomatisation translates into the latter. We provide generalisations of the presented rewrite rules, that can prove useful when trying to reduce terms in practice, and we show how to graphically make sense of these new rules. We show how to enrich the rewrite system to reach completeness for the dyadic fragments of quantum computation, used in particular in the Quantum Fourier Transform, and obtained by adding phase gates with dyadic multiples of π to the Toffoli-Hadamard gate-set. Finally, we show how to perform sums and concatenation of arbitrary terms, something which is not native in a system designed for analysing gate-based quantum computation, but necessary when considering Hamiltonian-based quantum computation.

5.13 A Profunctorial Semantics for Quantum Supermaps

Participant: Matt Wilson, James Hefford.

[10]

We develop a framework for studying higher-order processes over any theory of first-order maps. This framework allows for sequential (timelike) and parallel (spacelike) composition of first-order processes thereby capturing two key features expected in any spacetime. We then show that the higher-order maps give a good notion of transformation between timelike and spacelike arrangements of agents and in the particular case of quantum theory are precisely the “supermaps” studied in quantum foundations. The framework also allows one to differentiate between higher-order processes that possess a causal factorisation and those that do not.

6 Bilateral contracts and grants with industry

Quandela

Participants: Benoît Valiron, Pablo Arrighi, Nicolas Heurtel.

In the context of a PhD funded by CIFRE, QuaCS and Quandela are building a collaboration on the study of quantum linear optics. The approach is both theoretical –with the development of a formal language for reasoning on optical circuits, and practical, targeted towards simulation.

7 Partnerships and cooperations

7.1 International initiatives

7.1.1 Participation in other International Programs

QISS (John Templeton grant)

Participants: Pablo Arnault, Pablo Arrighi, Marc Baboulin, Marc de Visme, Benoît Valiron, Renaud Vilmart, Matt Wilson, Vladimir Zamdzhiev.

Title: The Quantum Information Structure of Spacetime

Partner Institutions:
- Institute for Quantum Optics and Quantum Information, Vienna
- Rotman Institute for Philosophy, Western University
- Center for Theoretical Physics, Aix-Marseille University
- Quantum Group and Clarendon Laboratory, University of Oxford
- Perimeter Institute
- University of Paris-Saclay, Quantum Computation Structures group
- Quantum Information and Computation Initiative, HKU
- Okinawa Institute of Science and Technology
- University of California Santa Barbara, Physics dpt
- Center for Quantum Information and Communication, Brussels
- Quantum Information Laboratory, Rome La Sapienza University
-
- Penn State University, Institute for Gravitation and the Cosmos
- Center for Mathematical Sciences, UNAM
- Bard College, New York
- ETH Zürich
- The University of Melbourne
- Royal Holloway, University of London
- Universität Bonn

Date: 2023-2026

Additionnal info:

QISS aims to found the physics of quantum spacetime on an information theoretical basis, bring within reach empirical access to quantum gravity phenomenology leveraging rapidly advancing quantum technologies, and promote interactions between physicists and philosophers. The broader scope of the consortium is to establish a long term research program that brings together the represented communities, towards the common goal of unravelling the Quantum Information Structure of Gravity.

QISS was initially conceived from interactions between the Quantum Gravity Group of the Center for Theoretical Physics and the Laboratory of Informatics and Systems at the University of Aix-Marseille, France. It was concretely conceived as an international collaborative project in fundamental research during an exploratory meeting held at the Slovak Academy of Sciences, Bratislava, in November 2018. QISS took off by a generous three year first phase grant by the John Templeton Foundation awarded to the Center for Space, Time and the Quantum that administers the project. A second phase grant was subsequently awarded to the QISS project by JTF.

7.1.2 Visits of international scientists

Vitor Fernandes

Status: PhD

Institution of origin: Universidade do Minho

Country: Portugal

Dates: March 2024 – July 2024

Context of the visit: Visiting Benoît Valiron, intenational advisor to Vitor's PhD

Mobility program/type of mobility: Research stay

Alejandro Díaz Caro

Visited institution: University of Buenos Aires

Country: Argentina

Dates: Nov 18, 2024 – Dec 8, 2024

Context of the visit: Research visit

Mobility program/type of mobility: Invited Professor (CentraleSupélec)

Masha Sosonkina

Visited institution: Old Dominion University, Norfolk

Country: USA

Dates: Sept 30, 2024 – Oct 4, 2024

Context of the visit: Research visit

Mobility program/type of mobility: US funding

Carla Ferradini

Status PhD

Institution of origin: ETH Zurich

Country: Switzerland

Dates: 27/06/2024 — 05/07/2024

Context of the visit: Collaboration on a project, and presentation of her research to the QuaCS team

Mobility program/type of mobility: Research stay

7.1.3 Visits to international teams

Pablo Arnault

Visited institution: Group “Informacion y computacion cuantica” (Physics department) of the Universidad Complutense, Madrid

Country: Spain

Dates: 23/02/2024

Context of the visit: Presentation of my work

Mobility program/type of mobility: Talk

Pablo Arnault

Visited institution: Instituto de Fisica Teorica (IFT) of the Universidad Autonoma, Madrid

Country: Spain

Dates: 15/04/2024 — 21/04/2024

Context of the visit: Presentation of my work and discussions with several members of the IFT

Mobility program/type of mobility: Research stay

Pablo Arnault

Visited institution: Instituto de Fisica Fundamental (IFF) of the CSIC, Madrid

Country: Spain

Dates: 16/04/2024

Context of the visit: Presentation of my work

Mobility program/type of mobility: Talk

Pablo Arnault

Visited institution: Luca Dellantonio’s group at the College of Engineering, Mathematics and Physical Sciences of the University of Exeter

Country: United Kingdom

Dates: 26/08/2024 — 30/08/2024

Context of the visit: Presentation of my work and discussion with Luca Dellantonio for potential collaboration

Mobility program/type of mobility: Research stay

Dogukan Bakircioglu

Visited institution: Luca Dellantonio’s group at the College of Engineering, Mathematics and Physical Sciences of the University of Exeter

Country: United Kingdom

Dates: 26/08/2024 — 30/08/2024

Context of the visit: Discussion with Luca Dellantonio for potential collaboration

Mobility program/type of mobility: Research stay

7.2 European initiatives

7.2.1 Horizon Europe

QCOMICAL

Participants: Pablo Arnault, Pablo Arrighi, Marc Baboulin, Marc de Visme, Benoît Valiron, Renaud Vilmart, Matt Wilson, Vladimir Zamdzhiev.

Title: Quantum Algorithms for Next Generation Aerospace Equipment

Partner Institutions:
- CENTRALESUPELEC, France
- UNIVERSITE PARIS-SACLAY, France
- Inria, France
- UNIVERSITE PARIS CITE, Fracne
- UNIVERSITE GRENOBLE ALPES, France
- UNIVERSITE D'AIX MARSEILLE, France
- UNIVERSITE PARIS-EST, France
- QUANDELA, France
- UNIVERSITA DI PISA, Italy
- Universita' degli Studi di Urbino Carlo Bo, Italy
- UNIVERSITA DEGLI STUDI DI CAGLIARI, Italy
- UNIVERSIDAD DE BUENOS AIRES, Argentina
- UNIVERSIDAD NACIONAL DE QUILMES, Argentina
- UNIVERSIDAD DE LA REPUBLICA, Uruguay

Date: 2024-2028

Additionnal info:

Quantum computing can be thought of in multiple ways. Among those ways, it can be seen as a computational model of quantum mechanics. Studying this model may have implications for our understanding of physics. It can also be seen as a new computational paradigm, with implications for computation, algorithms, and logic. Additionally, it can be viewed as a computational device that requires programming. Therefore, it is necessary to design and study programming languages for this purpose. The study of the foundations of quantum programming languages, type theory, and logic through the Curry-Howard correspondence may shed light on our understanding of quantum mechanics. Furthermore, it may lead to the development of new logics or the understanding of new structures in classical logic. Lastly, implementing these languages will enhance the way we program the new computers when they become widely used. In this project, we propose to study these various aspects of quantum computing, specifically focusing on the foundations of programming languages.

7.3 National initiatives

BPI-AeroQat

Participants: Pablo Arnault, Pablo Arrighi, Marc Baboulin, Marc de Visme, Benoît Valiron, Renaud Vilmart, Matt Wilson, Vladimir Zamdzhiev.

Title: Quantum Algorithms for Next Generation Aerospace Equipment

Partner Institutions:
- INRIA, France

- Alice & Bob, France
- Thales, France

Date: 2024-2027

Additionnal info:

Alice & Bob and Thales announced today a partnership to develop quantum algorithms capable of accelerating the simulation of aerospace equipment, such as radar or telecommunications antennas.

These algorithms will be developed for the new generation of "error resilient" quantum computers, known as FTQCs (Fault Tolerant Quantum Computers). Quantum computers can be classified as analog, noisy or fault tolerant. In line with the French government's recommendation that priority should be given to the latter, Alice & Bob and Thales are today joining forces to make France a frontrunner in the development of mature quantum technology.

The project evaluates whether these computers will be able to exponentially accelerate electromagnetic simulations, opening the door to the optimization of airborne equipment designs for the aerospace industry.

It will also assess accurately what resources will be needed, and therefore give a timeline on the time when quantum computers will be available to achieve this. Alice & Bob and Thales will work on the development of quantum algorithms for advanced electromagnetic simulation then test them on airborne equipment, such as radars and antennas. This will make it possible to estimate the exact number of qubits needed to industrialize and scale up these solutions.

The i-Démo Régions project, part of the France 2030 plan, has a budget of 2.6 million euros over 3 years, and has been accredited by the Systematic competitiveness cluster in the Paris region.

EPiQ (PEPR Quantique)

Participants: Pablo Arnault, Pablo Arrighi, Marc Baboulin, Marc de Visme, Benoît Valiron, Renaud Vilmart, Matt Wilson, Vladimir Zamdzhiev.

Title: Etude de la pile quantique : Algorithmes, modèles de calcul et simulation pour l'informatique quantique

Partner Institutions:
- INRIA, France
- CNRS, France
- CEA, France

Date: 2022-2027

Additionnal info:

The French quantum computing research community has always been at the forefront of international research. It thus provides the foundations for an ambitious strategy aiming at: (1) Understanding the advantages and limits of quantum computing via both quantum complexity research and the discovery and enhancement of algorithms (2) Defining the framework for quantum computation using high-level languages, comparison of computational models as well as using their relations for program optimization (3) Develop simulation tools to anticipate the performances of algorithms on noisy quantum machines. Algorithmic aspects are key in the field of quantum computing which witnesses a tremendous intensification of research efforts worldwide. Indeed, in addition to determining the design and the construction of hardware quantum processors, algorithms also constitute the interface through which users will solve their practical use cases leading to potential economic gain. Based on the outstanding French position, our project aims at developing algorithmic techniques for both noisy quantum machines (NISQ) and fault-tolerant ones so as to facilate their practical implementation. To this end, a first Work Package (WP) is dedicated to algorithmic techniques, a second one focuses on computational models and languages so as to facilitate the programming of quantum machines and to optimize the code execution steps. Lastly, the third WP aims at developing the simulation techniques of quantum computers.

EQIP (Inria challenge project)

Participants: Pablo Arnault, Pablo Arrighi, Marc Baboulin, Marc de Visme, Benoît Valiron, Renaud Vilmart, Matt Wilson, Vladimir Zamdzhiev.

Title: Engineering for Quantum Information Processors

Partner Institution:
- INRIA, France

Date: 2021-2024

Additionnal info:

While the technological development that has led us from the abacus to today's supercomputers or even to the latest achievements of machine learning are quite spectacular, one should not forget that they all fit the very same model of computation, formalized by Turing in the 1930s, and therefore fall under the umbrella of classical computing. Quantum physics has played a major role in this story through the 1st quantum revolution which gave birth to the transistor, the laser and the micro-processor. Rather surprisingly, the impact of quantum physics on the theory of computation is very likely still in its infancy. There is little doubt that an unprecedented shift will occur in the decades to come and that an entirely new form of computing will be dominant in 50 years (and probably much sooner). This is the object of the 2nd quantum revolution which will harness the quantum properties of matter and light to process data much more efficiently than is possible by purely classical means. The scope of applications remains hard to delineate at this point but covers a large spectrum of human activities: simulation of quantum systems will be crucial to develop new medicine, help fighting climate change by developing better materials to store or transport energy, reducing CO_2 emissions by developing efficient processes to capture CO_2; quantum computing will also be instrumental to solve optimization problems intractable today. At the same time, quantum technologies will dramatically impact cryptography and requires to implement important changes right now.
If the first glimpse of this second quantum revolution can be traced back to visionaries like Feynman or Deutsch in the early 80s, the fields of quantum computation and quantum simulation really took off in the last decade or so. The long-term objective of this line of work is to build a large universal quantum computer and the main scientific challenges today are to identify potential approaches for scaling up the small quantum processors consisting of a few tens of qubits already available, to anticipate how to program these new machines, and to understand what new capabilities will become accessible once quantum computing becomes available.

TaQC (ANR)

Participants: Pablo Arnault, Pablo Arrighi, Marc Baboulin, Marc de Visme, Benoît Valiron, Renaud Vilmart, Matt Wilson, Vladimir Zamdzhiev.

Title: Taming Quantum Causality

Partner Institutions:
- UPSaclay - LMF Université Paris-Saclay - Laboratoire Méthodes Formelles
- Inria Centre de Recherche Inria de Lyon - AT-LYS
- NEEL Institut Néel
- LARSIM Commissariat à l'énergie atomique et aux énergies alternatives

Date: 2023-2027

Additionnal info:

Quantum technologies provide advantages by exploiting non-classical resources, such as superposition or entanglement. Recently, it has been realized that one can obtain new advantages by exploiting causal structures that are inherently quantum. This quantum "causal indefiniteness" constitutes a novel resource and opens new perspectives in quantum information. Despite foundational progress and several experimental realizations, the concrete implications for quantum computing nevertheless remain poorly understood so far. In this project we will work to bridge this gap and to develop quantum causality as a new non-classical resource on par with superposition and entanglement.
To achieve this, we will develop three directions. (1) Firstly, we will develop a Generalised Probabilistic Theories approach to understand causal indefiniteness within a larger class of models. This will help clarify which causally indefinite process are physical and what precise role is played by genuinely non-classical resources such as superposition and entanglement. (2) Secondly, we will go beyond the standard example of the "quantum switch" to study more concrete models of causally indefinite computation. In doing so, we will systematically explore the possible applications of causal indefiniteness and unveil the potential of causally indefinite computations. (3) Finally, we will use ZX-Calculus to harness the capabilities of causal indefiniteness at the compilation level. Using ZX-Calculus as a springboard towards programming causally indefinite computations will help us optimize the use of this new resource.
Together, these goals work towards our ultimate objective of finding the right arguments in the right language to give causal indefiniteness a unique place among the leading conceptual and empirical paradigms of quantum information.

HQI (National Quantum Plan project)

Participants: Pablo Arnault, Pablo Arrighi, Marc Baboulin, Marc de Visme, Benoît Valiron, Renaud Vilmart, Matt Wilson, Vladimir Zamdzhiev.

Title: Hybrid HPC-Quantum platform and a research program

Partner Institutions:
- INRIA, France
- CNRS, France
- CEA, France
- GENCI, France
- France Universités, France
- ANR, France
- PIA4, France
- France Relance, France

Date: 2022-2027

Additionnal info:

In January 4th 2022, one year after the National Quantum Plan was announced by the French President, Neil Abroug, the national strategy coordinator, launched HQI: France Hybrid HPC-QC Initiative.

HQI is an integrated initiative. It combines a hybrid computing platform that couples several quantum processors with GENCI’s Joliot-Curie supercomputer hosted at TGCC (CEA), and an academic and industrial research program with user enablement.

This HPC-QC hybridization is an innovative and unique initiative that will benefit from the renowned expertise of the CEA/TGCC (Très Grand Centre de Calcul) teams in infrastructure operation, security and support to the scientific community.

The HQI initiative aims at serving the needs of French and European, academic and industrial research scientists, who want to evaluate free of charge, on a public infrastructure, the potential of quantum computing for their applications and develop international collaborations to foster open research.

8 Dissemination

8.1 Promoting scientific activities

8.1.1 Scientific events: organisation

General chair, scientific chair

- Vladimir Zamdzhiev for QPL 2024

Member of the organizing committees

- Pablo Arrighi for QPL 2024
- Renaud Vilmart for QPL 2024
- Benoît Valiron for CSL 2025, IWQC 2024, PlanQC 2025, and QPL 2024
- Vladimir Zamdzhiev for QPL 2024, POPL 2025 and ACT 2024

8.1.2 Journal

Member of the editorial boards

- Vladimir Zamdzhiev for the proceedings of QPL 2024 [12]
- Pablo Arrighi for TCS A
- Benoît Valiron for LMCS

Reviewer - reviewing activities

- Marc de Visme for MSCS
- Pablo Arrighi for Quantum and for Physical review letter
- Benoît Valiron for Quantum
- Renaud Vilmart for Physical revew letter

8.2 Conference

Reviewer - reviewing activities

- Marc de Visme for LICS
- Vladimir Zamdzhiev for LICS, OOPSLA
- Marc Baboulin for PPAM
- Renaud Vilmart for ACT, FSCD, MFCS, LICS, IJCAR, ReacTS

8.2.1 Invited talks

- “Fermions are qubit-local, invited talk at Foundations of quantum statistics”, Vienna, September 2024, by Pablo Arrighi.
- “Causality when geometry is quantum”, invited talk at IQSA, Brussels, August 2024, by Pablo Arrighi.
- “Bridging the gap between High Performance Computing and Quantum Computing”, TQCI, November 2024, EDF Lab, Saclay, by Marc Baboulin.

8.2.2 Leadership within the scientific community

Participant: Pablo Arrighi, Benoît Valiron, Marc Baboulin.

Quantum center of Saclay/Executive committee member (The center coordinates the French strategy for quantum technologies, in particular the QuantumEdu plan, at the scale of U. Paris-Saclay and Institut Polytechnique de Paris).

Participant: Pablo Arrighi.

Center for Quantum Spacetime/Board member The center gathers donations and manages grants in Quantum Gravity research, some at the interface with quantum information, e.g. QISS. Bureau du Comité des Équipes Projet/Member The BCEP evaluates the creation of new Inria and associated teams, as well as relationships with labs and doctoral schools.

Participant: Benoît Valiron.

Member of the working group opened by the AFNOR standards organization to harmonize quantum concepts and objects, in France and in interaction with ISO.

Participant: Benoît Valiron, Vladimir Zamdzhiev: .

IFIP Working Group on Foundations of Quantum Computation (WG 1.11 / 2.17)

Participant: Renaud Vilmart.

Co-supervisor of GT IQ (groupe de travail Informatique Quantique)

8.2.3 Scientific expertise

- Pablo Arrighi as grant reviewer
- Pablo Arrighi as “Shepherd” for the creation of the EPC Inria team Phiqus

8.2.4 Research administration

- Pablo Arrighi as REP of the EPC Inria QuaCS
- Benoît Valiron as co-REP of the EPC Inria QuaCS co-REP, and member of the special scientific council for the HCERES evaluation of the LMF
- Marc de Visme as member of the scientific council of the LMF, and member of the special scientific council for the HCERES evaluation of the LMF
- Renaud Vilmart as member of the scientific council of Inria Saclay

8.3 Teaching - Supervision - Juries

8.3.1 Teaching

Participant: Pablo Arnault.

- Recent trends in parallel, distributed, and quantum computing in M2-QDCS.
- Foundations of Quantum Information in M1-MPRI and M1-QDCS.

Participant: Marc de Visme: .

- Elements of computer science for quantum technologies. 25 students (Master, Arteq, ENS Paris-Saclay). Shared with Marc de Visme and Luc Lapointe.

Participant: Benoît Valiron.

- Various teaching at CentraleSupelec, for a total of 198h (equivalent TD).
- Introductory course on Quantum computation and Programming in M1-QDCS.
- Introductory course on Quantum computation and Programming in Arteq.

Participant: Marc Baboulin.

- Various teaching at Polytech Paris-Saclay.
- Initiation to quantum computing (5th year, Polytech Paris-Saclay).
- Simulation and synthesis of quantum circuits in M2-QDCS.

Participant: Vladimir Zamdzhiev.

- Introduction to Categories, M1 MPRI ENS Paris-Saclay
- Initiation to Research, M1 MPRI ENS Paris-Saclay

Participant: Renaud Vilmart.

- Elements of computer science for quantum technologies. 25 students (Master, Arteq, ENS Paris-Saclay). Shared with Marc de Visme and Luc Lapointe.
- Diagrammatic calculus and error correction. 20 students (Master 2 QDCS)
- Introduction to functional programming. Lab sessions, 25 students (L2 UFR sciences)

8.3.2 Supervision

- Dogukan Bakircioglu on Quantum simulation of quantum field theories, supervised by Pablo Arrighi, Pablo Arnault.
- Octave Mestoudjian on Generalised subsystems for quantum theory, supervised by Pablo Arrighi, Matt Wilson and Augustin Vanrietvelde.
- Marin Costes on Quantum networks theory, supervised by Pablo Arrighi and Luidnel Maignan.
- Nicolas Heurtel on Measurement-based quantum computing for photonics at Quandela (CIFRE), supervised by Pablo Arrighi, Benoît Valiron and Shane Mansfield.
- Thea Li on Operator Spaces and Models of Fragments of Linear Logic, supervised by Pablo Arrighi, Benoît Valiron and Vladimir Zamdzhiev.
- Louis Lemonnier on semantics of quantum control, supervised by Pablo Arrighi, Benoît Valiron and Vladimir Zamdzhiev.
- Julien Lamiroy on the connection between proof structures and superposition of execution, supervised by Benoît Valiron and Renaud Vilmart.
- Adham Zekri on compiling HHL on Toffoli+Hadamard, supervised by Benoît Valiron and Renaud Vilmart.
- Jérome Ricciardi on quantum program verification, supervised by Benoît Valiron and Christophe Chareton.
- Matthieu Robeyns on mixed-precision algorithms for low-rank matrix and tensor approximations, supervised by Marc Baboulin.
- Océane Koska on quantum algorithms for high performance, supervised by Marc Baboulin.
- Kinnari Dave on Combining Quantum and Classical Control, supervised by Romain Péchoux and Vladimir Zamdzhiev.

8.3.3 Juries

- Pablo Arrighi as jury member for Permanent researcher positions CRCN Inria Lyonjury member for Permanent researcher positions CRCN Inria Lyon
- Marc de Visme as jury member for the ENS' written competitive exam (grading the fundamental computer science written exam)
- Benoît Valiron as jury member for the PhD thesis of Antoine Henry
- Marc Baboulin as reviewer and jury member for the PhD thesis of Romeo Molina (LIP6)
- Marc Baboulin as jury member for the PhD thesis of Matthieu Robeyns (LISN)

8.4 Popularization

8.4.1 Participation in Live events

- "Quantum cellular automata: structure, universality, applications to quantum simulation", October 2024, Escuela Colombiana de Ingeniería (ECI) and Universidad Nacional de Colombia, by Pablo Arrighi
- "Programmer dans les espaces vectoriels des structures de données quantiques", September 2024, ENS Rennes, by Benoît Valiron.
- "Why we care about quantum computing, and how we deal with it, IRT SystemX, Paris Saclay", by Renaud Vilmart.

9 Scientific production

9.1 Major publications

[1] A. Clément, N. Heurtel, S. Mansfield, S. Perdrix and B. Valiron. 'A Complete Equational Theory for Quantum Circuits'. In: *38th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*. 2023 38th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS). Boston, United States: IEEE, July 2023, pp. 1–13. DOI: 10.1109/LICS56636.2023.10175801. URL: https://hal.science/hal-03926757.

[2] N. Eon, G. D. Molfetta, G. Magnifico and P. Arrighi. 'A relativistic discrete spacetime formulation of 3+1 QED'. In: *Quantum* 7 (8th Nov. 2023), p. 1179. DOI: 10.22331/q-2023-11-08-1179. URL: https://hal.science/hal-03944082.

9.2 Publications of the year

International journals

[3] P. Arrighi, G. Dowek and A. Durbec. 'Time arrow without past hypothesis: a toy model explanation'. In: *New Journal of Physics* 26.11 (28th Nov. 2024), p. 113019. DOI: 10.1088/1367-2630/ad93f5. URL: https://hal.science/hal-04856924 (cit. on p. 6).

[4] P. Arrighi, A. Durbec and M. Wilson. 'Quantum networks theory'. In: *Quantum* 8.1508 (2024). DOI: 10.22331/q-2024-10-23-1508. URL: https://hal.science/hal-04727044 (cit. on p. 6).

[5] R. Vilmart. 'Rewriting and Completeness of Sum-Over-Paths in Dyadic Fragments of Quantum Computing'. In: *Logical Methods in Computer Science* 20.1 (2024). DOI: 10.46298/LMCS-20(1:20)2024. URL: https://hal.science/hal-04496100 (cit. on p. 9).

International peer-reviewed conferences

[6] P. Arrighi, G. Dowek and A. Durbec. 'A toy model provably featuring an arrow of time without past hypothesis'. In: *LNCS*. RC 2024 - 16th International Conference on Reversible Computation. Vol. 14680. Lecture Notes in Computer Science. Torun, Poland: Springer Nature Switzerland, 29th May 2024, pp. 50–68. DOI: 10.1007/978-3-031-62076-8_4. URL: https://hal.science/hal-04727052 (cit. on p. 6).

[7] K. Chardonnet, L. Lemonnier and B. Valiron. 'Semantics for a Turing-Complete Reversible Programming Language with Inductive Types'. In: *9th International Conference on Formal Structures for Computation and Deduction (FSCD 2024). Leibniz International Proceedings in Informatics (LIPIcs)*. FSCD 2024 - 9th International Conference on Formal Structures for Computation and Deduction. Vol. 299. 19. Tallinn, Estonia: Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2024, 19:1–19:19. DOI: 10.4230/LIPIcs.FSCD.2024.19. URL: https://hal.science/hal-04636603 (cit. on p. 7).

[8] A. Clément, N. Delorme and S. Perdrix. 'Minimal Equational Theories for Quantum Circuits'. In: LICS '24: 39th Annual ACM/IEEE Symposium on Logic in Computer Science. Proceedings of the 39th Annual ACM/IEEE Symposium on Logic in Computer Science 27. Tallinn, Estonia: ACM, 8th July 2024, pp. 1–14. DOI: 10.1145/3661814.3662088. URL: https://hal.science/hal-04399210 (cit. on p. 8).

[9] A. Clément, N. Delorme, S. Perdrix and R. Vilmart. 'Quantum Circuit Completeness: Extensions and Simplifications'. In: International Conference on Computer Science Logic CSL 2024. Naples, Italy, Feb. 2024. DOI: 10.4230/LIPIcs.CSL.2024.20. URL: https://hal.science/hal-04016498 (cit. on p. 7).

[10] J. Hefford and M. Wilson. 'A Profunctorial Semantics for Quantum Supermaps'. In: *LICS '24: 39th Annual ACM/IEEE Symposium on Logic in Computer Science*. LICS 2024 - 39th Annual ACM/IEEE Symposium on Logic in Computer Science. Tallinn, Estonia: ACM, 2024, p. 43. DOI: 10.1145/3661814.3662123. URL: https://hal.science/hal-04720947 (cit. on p. 9).

[11] B. Valiron, M. de Visme and V. Fernandes. 'Non-deterministic, probabilistic, and quantum effects through the lens of event structures'. In: The 22nd Asian Symposium on Programming Languages and Systems. Kyoto, Japan, Oct. 2024. URL: https://hal.science/hal-04744478 (cit. on p. 8).

Edition (books, proceedings, special issue of a journal)

[12] A. Díaz-Caro and V. Zamdzhiev, eds. *Proceedings of the 21st International Conference on Quantum Physics and Logic*. Quantum Physics and Logic. Vol. 406. EPTCS, 9th Aug. 2024. DOI: 10.4204/EPTCS.406. URL: https://hal.science/hal-04736872 (cit. on p. 17).

Doctoral dissertations and habilitation theses

[13] B. Valiron. 'On Quantum Programming Languages'. Université Paris Saclay, 24th Sept. 2024. URL: https://theses.hal.science/tel-04740855.

Reports & preprints

[14] B. Lindenhovius and V. Zamdzhiev. *The Category of Operator Spaces and Complete Contractions*. 30th Dec. 2024. URL: https://inria.hal.science/hal-04880599 (cit. on p. 7).

[15] U. Nzongani and P. Arnault. *Adjustable-depth quantum circuit for position-dependent coin operators of discrete-time quantum walks*. 16th Jan. 2024. URL: https://hal.science/hal-04396459 (cit. on pp. 8, 9).

[16] U. Nzongani, N. Eon, I. Márquez-Martín, A. Pérez, G. Di Molfetta and P. Arrighi. *Dirac quantum walk on tetrahedra*. 2024. DOI: 10.48550/arXiv.2404.09840. URL: https://hal.science/hal-04727049 (cit. on p. 9).

[17] S. Ty, R. Vilmart, A. Tahmasebimoradi and C. Mang. *Double-Logarithmic Depth Block-Encodings of Simple Finite Difference Method's Matrices*. 2024. DOI: 10.48550/arXiv.2410.05241. URL: https://hal.science/hal-04725154 (cit. on p. 8).

[18] R. Vilmart and M. de Visme. *Minimality in Finite-Dimensional ZW-Calculi*. 29th Jan. 2024. URL: https://hal.science/hal-04496193 (cit. on p. 6).

9.3 Cited publications

[19] P. Arnault, P. Arrighi, S. Herbert, E. Kasnetsi and T. Li. *A typology of quantum algorithms*. 2024. arXiv: 2407.05178 [quant-ph]. URL: https://arxiv.org/abs/2407.05178 (cit. on p. 8).

[20] O. Koska, M. Baboulin and A. Gazda. 'A Tree-Approach Pauli Decomposition Algorithm with Application to Quantum Computing'. In: *ISC High Performance 2024 Research Paper Proceedings (39th International Conference)*. 2024, pp. 1–11. DOI: 10.23919/ISC.2024.10528938 (cit. on p. 7).

[21] U. Nzongani, J. Zylberman, C.-E. Doncecchi, F. Debbasch and P. Arnault. 'Quantum circuits for discrete-time quantum walks with position-dependent coin operator JO - Quantum Information Processing'. In: *Quantum Information Processing* 22 (2023). DOI: 10.1007/s11128-023-03957-8 (cit. on pp. 8, 9).